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A New Method for Evaluating Two Dimensional Line Integrals Using Maple

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Abstract: **In this article, we study two types of two dimensional line integral problems. The closed forms of the two types of line integrals can be determined by using a complex integral formula. In addition, we provide two examples to do calculation practically. The research method used in this paper is to get the results by hand first, and then calculate the approximate values of the answers by Maple.**

Keyword: **Line integrals, Complex integral formula, Closed forms, Maple.**

I. INTRODUCTION

The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. Moreover, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests.

In calculus and engineering mathematics, there are many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. In this paper, we study the following two types of two dimensional line integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int_{\gamma} \frac{1}{2}a\alpha x^{\alpha-1} \ln\left[(ax^{\alpha}+c)^2+(by^{\beta}+d)^2\right]dx - b\beta y^{\beta-1}\arctan\left(\frac{by^{\beta}+d}{ax^{\alpha}+c}\right)dy\,, \tag{1}$$

and

$$\int_{\gamma} a\alpha x^{\alpha-1}\arctan\left(\frac{by^{\beta}+d}{ax^{\alpha}+c}\right)dx+\frac{1}{2}b\beta y^{\beta-1}\ln\left[(ax^{\alpha}+c)^2+(by^{\beta}+d)^2\right]dy\,. \tag{2}$$

Where $a, b, c, d, \alpha, \beta$ are real numbers, and $\gamma: [t_1, t_2] \to R^2$ is a piecewise smooth curve in R^2 defined by $\gamma(t) = (x(t), y(t))$ which satisfies $ax(t)^{\alpha} + c \neq 0$ for all $t \in [t_1, t_2]$. The closed forms of the two types of line integrals can be determined by using a complex integral formula; these are the major results of this article (i.e., Theorems 1 and 2). Adams et al. [1], Nyblom [2], and Oster [3] studied some integral problems. Moreover, Yu [10], Yu and Huang [9], and Chen and Yu [11] used some techniques to solve the line integral problems. On the other hand, Yu [4-8], Yu and Chen [12], and Yu and Sheu [13] provided some new methods to evaluate the integral problems, which including complex power series, integration term by term theorem, Parseval's theorem, area mean value theorem, and generalized Cauchy integral formula. In this study, we also propose some examples to demonstrate the manual calculations, and verify the results using Maple.

II. PRELIMINARIES AND MAIN RESULTS

At first, we introduce some definitions used in this article.

2.1 Definitions:

The complex logarithmic function $f(z) = lnz$ is defined by $lnz = ln|z| + i\theta$, where $i = \sqrt{-1}$, z is a non-zero complex number, θ is a real number, $z = |z| \cdot e^{i\theta}$, and $-\pi < \theta \leq \pi$.

To obtain the major results, the following lemma is needed which is a complex integral formula used in this paper.

Lemma 1 *Suppose that λ, C are constant complex numbers, then the complex integral*

$$\int ln(z+\lambda)\,dz = (z+\lambda)\,ln(z+\lambda) - z + C. \tag{3}$$

Proof. $\int ln(z+\lambda)\,dz = \int ln\,u\,du$ (where $u = z + \lambda$)

$$= u \cdot lnu - u + C_1$$

$$= (z+\lambda)\,ln(z+\lambda) - z + C.$$ Q.e.d.

Next, we obtain the closed forms of the line integrals (1) and (2).

Theorem 1 *Assume that $a, b, c, d, \alpha, \beta$ are real numbers, and $\gamma: [t_1, t_2] \to R^2$ is a piecewise smooth curve in R^2 defined by $\gamma(t) = (x(t), y(t))$ which satisfies $ax(t)^\alpha + c \neq 0$ for all $t \in [t_1, t_2]$. Then the line integral*

$$\int_\gamma \frac{1}{2}a\alpha x^{\alpha-1}\,ln\left[(ax^\alpha+c)^2+(by^\beta+d)^2\right]dx - b\beta y^{\beta-1}arctan\left(\frac{by^\beta+d}{ax^\alpha+c}\right)dy$$

$$= F(x(t_2), y(t_2)) - F(x(t_1), y(t_1)) \text{ , where}$$

$$F(x,y) = \frac{1}{2}(ax^\alpha+c)\cdot ln\left[(ax^\alpha+c)^2+\left(by^\beta+d\right)^2\right] - \left(by^\beta+d\right)\cdot arctan\left(\frac{by^\beta+d}{ax^\alpha+c}\right) - ax^\alpha. \tag{4}$$

Proof. In Lemma 1, taking $z = ax^\alpha + iby^\beta$, $\lambda = c + id$, we have

$$\int ln\left[(ax^\alpha+c)+i\left(by^\beta+d\right)\right]d(ax^\alpha+iby^\beta)$$

$$= \left[(ax^\alpha+c)+i\left(by^\beta+d\right)\right]\cdot ln\left[(ax^\alpha+c)+i\left(by^\beta+d\right)\right] - \left(ax^\alpha+iby^\beta\right) + C. \tag{5}$$

It follows from the definition of logarithmic function that

$$\int\{\frac{1}{2}ln\left[(ax^\alpha+c)^2+\left(by^\beta+d\right)^2\right] + iarctan\left(\frac{by^\beta+d}{ax^\alpha+c}\right)]\}(a\alpha x^{\alpha-1}dx + ib\beta y^{\beta-1}dy)$$

$$= \left[(ax^\alpha+c)+i\left(by^\beta+d\right)\right]\cdot\{\frac{1}{2}ln\left[(ax^\alpha+c)^2+\left(by^\beta+d\right)^2\right] + iarctan\left(\frac{by^\beta+d}{ax^\alpha+c}\right)]\} - \left(ax^\alpha+iby^\beta\right) + C. \tag{6}$$

And hence, using the equality of real parts of both sides of Eq. (6), we obtain the desired result. Q.e.d.

Theorem 2 *Suppose that the assumptions are the same as Theorem 1, then the line integral*

$$\int_\gamma a\alpha x^{\alpha-1}arctan\left(\frac{by^\beta+d}{ax^\alpha+c}\right)dx + \frac{1}{2}b\beta y^{\beta-1}ln\left[(ax^\alpha+c)^2+(by^\beta+d)^2\right]dy$$

$$= G(x(t_2), y(t_2)) - G(x(t_1), y(t_1)) \text{ , where}$$

$$G(x,y) = \frac{1}{2}\left(by^\beta+d\right)\cdot ln\left[(ax^\alpha+c)^2+\left(by^\beta+d\right)^2\right] + (ax^\alpha+c)\cdot arctan\left(\frac{by^\beta+d}{ax^\alpha+c}\right) - by^\beta. \tag{7}$$

Proof. By the equality of imaginary parts of both sides of Eq. (6), the desired result is obtained. Q.e.d.

III. TWO EXAMPLES

For the two dimensional line integral problems discussed in this paper, two examples are provided and we use Theorems 1 and 2 to determine their closed forms. Moreover, Maple is used to calculate the approximate values of the line integrals and their solutions for verifying our answers.

Example 1. In Theorem 1, if taking $a = 2, b = 1, c = 1, d = 2, \alpha = 1, \beta = 1,$ $x(t) = t$ and $y(t) = 2t, t \in [0,1]$, then we have

$$\int_0^1 [ln(8t^2 + 12t + 5) - 2 \cdot arctan\left(\frac{2t+2}{2t+1}\right)]dt$$

$$= \frac{5}{2} ln5 + 2arctan2 - 4arctan\frac{4}{3} - 2. \quad (8)$$

Using Maple to calculate the approximations of both sides of Eq. (8) as follows:

>evalf(int(ln(8*t^2+12*t+5)-2*arctan((2*t+2)/(2*t+1)),t=0..1),18);

0.52871134466698300

>evalf(5/2*ln(5)+2*arctan(2)-4*arctan(4/3)-2,18);

0.52871134466698300

Example 2. Let $a = 2, b = 1, c = 3, d = 1, \alpha = 2, \beta = 2,$ $x(t) = 2t$ and $y(t) = 3t, t \in [1,2]$ in Theorem 2, then

$$\int_1^2 [16t \cdot arctan\left(\frac{9t^2+1}{8t^2+3}\right) + 9t \cdot ln(145t^4 + 66t^2 + 10)]dt$$

$$= \frac{37}{2} \cdot ln(2594) + 35arctan\left(\frac{37}{35}\right) - 5 \cdot ln(221) - 11 \cdot arctan\left(\frac{10}{11}\right) - 27. \quad (9)$$

We also use Maple to obtain the approximate values of both sides of Eq. (9).

>evalf(int(16*t*arctan((9*t^2+1)/(8*t^2+3))+9*t*ln(145*t^4+66*t^2+10),t=1..2),18);

111.781821580979749

>evalf(37/2*ln(2594)+35*arctan(37/35)-5*ln(221)-11*arctan(10/11)-27,18);

111.781821580979749

IV. CONCLUSION

As mentioned, we mainly use a complex integral formula to find the closed forms of two types of two dimensional line integrals. In fact, the applications of complex integral formulas are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and use Maple to verify our answers. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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